

## Phase breaking in ballistic quantum dots: Experiment and analysis based on chaotic scattering

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We introduce two methods of extracting the rate of loss of electron phase coherence in ballistic quantum dot microstructures from statistical measures of magnetic-field-induced conductance fluctuation. The first method uses changes in the (magnetic-field) power spectrum of conductance fluctuations upon changing the average conductance through the dot. The second, based on a Hauser-Feshbach-like formula from nuclear physics, relates the phase-breaking rate to the amplitude of the conductance fluctuations. The two methods are then applied to experimental data from GaAs/Al<sub>x</sub>Ga<sub>1-x</sub>As quantum dots in the shape of a stadium billiard. Preliminary results give phase-breaking rates that are consistent with those previously found in quasiballistic GaAs/Al<sub>x</sub>Ga<sub>1-x</sub>As microstructures.

### I. INTRODUCTION

Electron transport in high-mobility semiconductor microstructures at low temperatures may simultaneously reveal evidence of quantum phase coherence as well as the ballistic motion of electrons. Phase coherence gives rise to interference phenomena similar to those observed in disordered mesoscopic systems, while ballistic electron motion—by definition, occurring when the elastic mean-free path exceeds the device size—implies that essentially all large-angle scattering occurs as reflection from the walls of the device, so that transport becomes sensitive to the shape of the microstructure.<sup>1-4</sup> In addition, transport in microstructures may exhibit quantization effects as feature sizes approach the electron Fermi wavelength,  $\lambda_F \sim 40$  nm.<sup>5</sup> Given these properties, ballistic microstructures are well suited to exploring the common ground between three related areas of physics:<sup>6</sup> chaotic scattering and its quantum manifestations,<sup>7</sup> conductance fluctuations in mesoscopic systems,<sup>8</sup> and the statistical theory of nuclear scattering.<sup>9</sup> In this paper, we use some newly recognized connections between these areas to derive two simple methods of extracting the rate of loss of electron phase coherence  $\gamma_\varphi$  ( $=1/\tau_\varphi$ ) from magnetotransport measurements in ballistic microstructures. The arguments we use extend recent semiclassical treatments of chaotic scattering in ballistic microstructures.<sup>10,11</sup>

We then use these methods to measure  $\gamma_\varphi$  at millikelvin temperatures in gate-defined GaAs/Al<sub>x</sub>Ga<sub>1-x</sub>As quantum dots fabricated in the shape of a stadium billiard with quantum point-contact leads [Fig. 1(a)]. The values we find for  $\gamma_\varphi$  have large uncertainties due to the narrow range of parameters available in the devices studied, which were not optimized for these measurements. For this reason, the measurements reported here should be viewed as a demonstration of a novel technique rather

than an exhaustive experimental investigation of phase breaking. Nonetheless, the values found for  $\gamma_\varphi$  are consistent with other recently reported measurements of low-temperature phase-breaking rates in quasiballistic GaAs/Al<sub>x</sub>Ga<sub>1-x</sub>As microstructures at low temperatures.<sup>12-14</sup> In these previous studies, phase-breaking rates were inferred from weak localization<sup>12,13</sup> and conductance fluctuation<sup>14</sup> measurements using formulas that assume diffusive transport in at least one spatial direction. Results obtained in Refs. 12-14 suggest that low-temperature phase-breaking rates in quasiballistic microstructures exceed the theoretically predicted rate for diffusive systems<sup>15</sup> (if one assumes that the true electron-gas temperature is equal to the measured refrigerator temperature), and that these rates become independent of

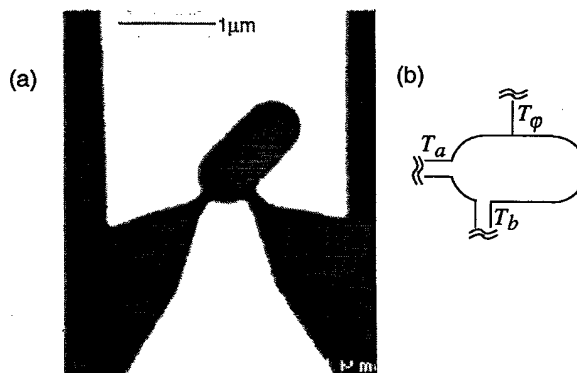


FIG. 1. (a) Electron micrograph of stadium quantum dot. The light region is gate metal on a GaAs heterostructure surface. 1- $\mu$ m bar shown for scale. (b) Schematic "effective lead" representation of the phase-breaking channel with transmission probability  $T_\varphi$ .

temperature below about 0.5 K. It is difficult to rule out spurious heating of the electron gas and other nonequilibrium effects as sources of disagreement with the theory. Unfortunately, no detailed theory of low-temperature phase breaking (comparable, for example, to Ref. 15) exists for ballistic quantum dots, i.e., microstructures smaller than the electron mean free path in all spatial directions.

## II. EXPERIMENTAL DETAILS

The quantum dot stadia we have investigated are defined on the surface of a GaAs/Al<sub>0.3</sub>Ga<sub>0.7</sub>As heterostructure using Cr/Au (20 Å/80 Å) electrostatic gates patterned via electron-beam lithograph and liftoff [Fig. 1(a)].<sup>4</sup> We report measurements on two stadium structures, denoted stadium 1 and 2, fabricated on nominally identical wafers, each with an accompanying circular quantum dot next to it. A mobility  $\mu$  of 265 000 cm<sup>2</sup>/Vs and a sheet density  $n$  of  $3.8 \times 10^{11}$  cm<sup>-2</sup> were measured in both samples using a van der Pauw geometry with all gates shorted to the electron gas at a refrigerator temperature of 16 mK with the samples cooled in the dark. Also, a slightly lower sheet density of  $3.6 \times 10^{11}$  cm<sup>-2</sup> within the dot was inferred from modulations in the amplitude of Aharonov-Bohm-like oscillations observed at magnetic fields above 1 T. These amplitude modulations are periodic in  $1/B$  and presumably correspond to Landau levels in the dot passing through the Fermi surface, and so provide a measure of the density within the dot. These values give an elastic (transport) mean free path of  $l_e = 2.6 \mu\text{m}$  and an elastic-scattering time of  $\tau = 10$  ps.

The size of each stadium dot is  $1.2 \times 0.5 \mu\text{m}^2$ , giving an effective area of  $\sim 0.5 \mu\text{m}^2$ , after accounting for a 50-nm edge depletion. The stadium is connected to the bulk two-dimensional electron gas via a pair of point-contact leads oriented at 90°. Each lead has a lithographic width of 0.14  $\mu\text{m}$  and passes at most  $\sim 3$  lateral subbands. (We use the difference between the lithographic lead width and the actual lead width, inferred from conductance, to estimate the 50-nm edge depletion.) Conductances of the leads can be controlled by small changes in gate voltage, which only slightly affect the size and shape of the stadium. Magnetotransport measurements were carried out in a dilution refrigerator with mixing-chamber temperatures down to 16 mK using standard ac lock-in techniques with an 11-Hz current bias. The voltage drop across the sample was kept below 2  $\mu\text{V}$  in all runs to avoid excessive heating of the electrons.

## III. PHASE-BREAKING RATE FROM POWER SPECTRUM OF CONDUCTANCE FLUCTUATIONS

In order to extract a phase-breaking rate from conductance fluctuation statistics, we begin by considering the classical problem of escape from an open two-dimensional billiard such as our two-lead stadium dot. If the motion of particles bouncing in the billiard is chaotic, then the escape probability per unit time will be constant, and we may define an escape rate  $\gamma_{\text{esc}}$ . That is, the classical probability for an electron to remain trapped within

the device falls off exponentially in time,  $P_{\text{esc}}(t) = P_0 e^{-\gamma_{\text{esc}} t}$ . This will hold as long as the time  $t$  is not too short, i.e., more than a few wall bounces.<sup>16</sup> Semiclassically, the escape rate  $\gamma_{\text{esc}}$  can be expressed in terms of the lead widths  $w_a$  and  $w_b$ , the area  $A$  of the device, and the electron velocity  $v_F = 2\pi\hbar/m^* \lambda_F$  as follows:  $\gamma_{\text{esc}} \approx (w_a + w_b)v_F/\pi A$ .<sup>10,17,18</sup>

In the semiclassical picture, electrons follow classical trajectories but have associated with them a phase which allows for quantum interference. The phase along the trajectory advances by  $2\pi$  in a Fermi wavelength  $\lambda_F$ , and also couples to an applied magnetic field  $B$  though its vector potential, giving a phase shift proportional to  $B$  times the area swept out by the trajectory perpendicular to  $B$ . The number of occupied modes  $\Lambda$  in each lead is given by the number of half Fermi wavelengths across the opening,  $\Lambda_a = 2w_a/\lambda_F$  (for lead  $a$ ). This semiclassical picture is appropriate when (i) the lateral dimensions of the device are much greater than  $\lambda_F$ , (ii) there are many modes in the leads,  $\Lambda_a, \Lambda_b \gg 1$ , and (iii) all modes are fully transmitting,  $T_a^i = T_b^i = 1$ , where  $T_a^i$  is the transmission coefficient of the  $i$ th mode in lead  $a$ . Partial transmission may be included by defining a total transmission probability  $T_a = \sum_i \Lambda_a T_a^i$  for lead  $a$  and similarly for lead  $b$ . Note that when all  $T_{a(b)}^i = 1$ , we recover  $T_{a(b)} = \Lambda_{a(b)}$ . In terms of  $T_a$  and  $T_b$ , the escape rate can be written  $\hbar\gamma_{\text{esc}} \approx (T_a + T_b)\Delta/2\pi$  where  $\Delta$  is the mean spacing of (spin-degenerate) energy levels in the dot,  $\Delta = 2\pi\hbar^2/m^* A$ . For our structures,  $\Delta \sim 14 \mu\text{eV}$ .

Quantum interference of electrons traversing the dot will be destroyed by scattering events that randomize the phase of the electrons. Phase-breaking scattering like chaotic escape may be characterized by a rate  $\gamma_\varphi$  ( $=1/\tau_\varphi$ ) which we assume to be independent of  $\gamma_{\text{esc}}$ . Then, because only electrons which have not suffered phase breaking will contribute to the fluctuating part of the conductance, one can define a total effective loss rate of the phase-coherent electrons as the sum of these rates,

$$\gamma_{\text{eff}} = \gamma_\varphi + \gamma_{\text{esc}} \quad (1)$$

It is this effective rate,  $\gamma_{\text{eff}}$ , that we will use to characterize the statistics of magnetoconductance fluctuations. This simple approach strongly resembles the method introduced by Büttiker<sup>19,20</sup> of adding an imaginary extra lead to a device which draws no current but provides a channel for phase breaking. The correspondence to Büttiker's "effective lead" model can be made explicit by assigning a transmission coefficient  $T_\varphi = 2\pi(\hbar\gamma_\varphi/\Delta)$  to the phase-breaking channel. In this formulation, the effective total number of channels through which a phase-coherent electron may "exit" the device—including by undergoing phase breaking—is

$$T_{\text{eff}} = (T_a + T_b + T_\varphi) = 2\pi \frac{\hbar\gamma_{\text{eff}}}{\Delta} \quad (2)$$

Formulas similar to Eq. (2) may be found in the nuclear physics literature, dating back to the work of Bohr.<sup>21</sup> An effective channel approach has also recently been applied to the problem of chaotic scattering in microwave sys-

tems to account for absorption by the walls of the cavity.<sup>22</sup> We emphasize that the effective channel model is applicable to ballistic transport only insofar as escape through the leads is an exponential process. In certain nonchaotic billiards, for instance, the escape probability obeys a power law.<sup>23,24</sup> For ballistic microstructures with such nonchaotic shapes, phase breaking and exiting through the leads are not on equal footing, and a simple effective channel model is not appropriate.

Because our control parameter is magnetic field, we must relate the effective escape rate in Eq. (1) to the distribution of areas cut by electrons traversing the structure. Arguments presented by Jensen<sup>17</sup> and others,<sup>10,11</sup> as well as numerical evidence,<sup>10,11</sup> indicate that classically, the distribution of areas cut by trajectories within a chaotic billiard is exponential,  $P(A) = P(0)\exp(-2\pi\alpha|A|)$ , with a characteristic inverse area  $\alpha$  that is related to the escape rate according to  $\alpha \sim k\sqrt{\gamma_{\text{esc}}}$ , where  $k$  is a geometry-dependent factor. We will introduce phase breaking into this classical expression in a simple, physically reasonable way: by replacing the (classical) escape rate  $\gamma_{\text{esc}}$  with the effective escape rate  $\gamma_{\text{eff}}$  of the un-phase-broken electrons, with the corresponding change to the distribution of areas cut by these electrons.

Based on the above form for  $P(A)$ , semiclassical analysis,<sup>10,11</sup> yields a Lorentzian-squared form for the autocorrelation of the conductance fluctuations:  $C(\Delta B) = \langle \delta g(B)\delta g(B + \Delta B) \rangle = C(0)/[1 + (\Delta B/\alpha\phi_0)^2]^2$ . In this expression  $\delta g(B) = [g(B) - \langle g \rangle]$  is the fluctuation in conductance away from the average conductance  $\langle g \rangle$ , in units of  $e^2/h \approx (25.8 \text{ k}\Omega)^{-1}$ . Angular brackets denote magnetic-field averages, taken over a field range much greater than the characteristic field of the fluctuations.  $\phi_0 = h/e = 4.14 \times 10^{-3} \text{ T}\mu\text{m}^2$  is the quantum of flux. The Fourier transform  $F$  of  $C(\Delta B)$  gives the magnetic power spectrum  $S_g(f)$  of the conductance fluctuations,

$$S_g(f) = \text{F.T.}[C(\Delta B)] = S_g(0)[1 + (2\pi\alpha\phi_0)f]^2 e^{-2\pi\alpha\phi_0 f}, \quad (3)$$

where  $f$  is the magnetic frequency in units of cycles/T. The semiclassical derivation leading to Eq. (3) only applies in the regime of overlapping resonance,  $\hbar\gamma_{\text{eff}} > \Delta$ .<sup>10,11</sup> Note that in the present formulation, this resonance width *includes* the contribution of phase breaking.

If we now assume that electrons in the stadium dot undergo several wall bounces before exiting and move ergodically within the dot, the average conductance will be related to the lead transmissions  $T_a$  and  $T_b$  by the simple resistors-in-series form  $\langle g \rangle = 2(1/T_a + 1/T_b)^{-1}$ , where the factor of 2 accounts for spin degeneracy.<sup>6,20,25</sup> Note that  $\langle g \rangle$  in the above expression is independent of  $T_\varphi$ . For equivalent leads,  $T_a = T_b = T$ , the average conductance across the device is simply  $\langle g \rangle = T$ .

With the above discussion as background, we are now ready to describe the first set of measurements to extract  $\gamma_\varphi$ . The idea is to compare magnetic power spectra  $S_g(f)$  measured in a given device during the same cooldown at

different gate voltages, such that in one case the average conductivity through the device is high,  $\langle g \rangle_H \sim 2.7$ , and in the other case it is low,  $\langle g \rangle_L \sim 1.1$ . For each run, conductance fluctuations  $\delta g(B)$  are extracted from a slowly varying average conductance by subtracting a low-order polynomial fit. Then, averaged power spectra of  $\delta g(B)$  are computed for the high- and low-conductivity data, and characteristic inverse areas,  $\alpha_H$  and  $\alpha_L$ , are found by fitting Eq. (3) to each spectrum. Assuming  $T_a \approx T_b$ , and using the scaling  $\gamma_{\text{eff}} \propto \alpha^2$ , we may extract  $T_\varphi$  using the equality (independent of the geometrical factor  $k$ ),

$$\frac{\langle g \rangle_H + T_\varphi/2}{\langle g \rangle_L + T_\varphi/2} = \left[ \frac{\alpha_H}{\alpha_L} \right]^2, \quad (4)$$

and finally find the phase-breaking rate from the relation  $\gamma_\varphi = (\Delta/2\pi\hbar)T_\varphi$ .

Fits to the spectra for stadium 1, shown in Fig. 2, yield  $\alpha_H = 0.95 \pm 0.02 \mu\text{m}^{-2}$  and  $\alpha_L = 0.77 \pm 0.02 \mu\text{m}^{-2}$  with average low-field conductances of  $\langle g \rangle_H = 2.7$  and  $\langle g \rangle_L = 1.1$ . From these values and Eq. (4), we find an

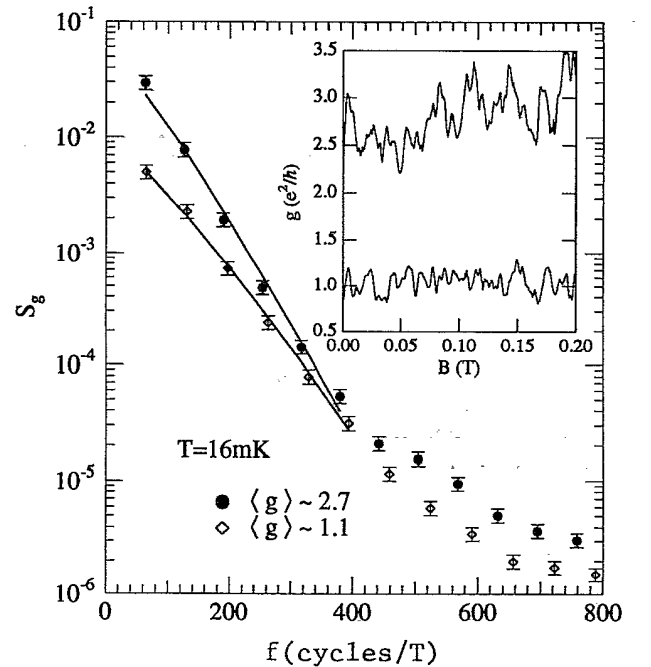


FIG. 2. Averaged power spectra  $S_g(f)$  of magnetoconductance fluctuations of stadium 1 at 16 mK for low (diamond) and high (dot) average conductance. The two runs were measured during the same cooldown, at gate voltages of  $-0.57 \text{ V}$  ( $-0.44 \text{ V}$ ) for low (high) conductance. The spectra have different slopes at low magnetic frequency  $f$ , indicating higher total escape rate at large average conductance. Solid curves are two-parameter fits to the semiclassical prediction, Eq. (3), over the range  $f \sim 64\text{--}400$  cycles/T, where the spectra appear to be well described by the form of Eq. (3). Each averaged spectrum is computed by summing individual fast-Fourier-transform power spectra of  $\delta g(B)$  from half-overlapping 128-point blocks covering  $\sim 15 \text{ mT}$  each, over the range  $|B| < 0.25 \text{ T}$ . The inset shows a segment of conductance fluctuation data for low and high average conductance.

effective number of phase-breaking channels  $T_\varphi = 4 \pm 1$ , giving a phase-breaking rate  $\gamma_\varphi \sim (1.3 \pm 0.4) \times 10^{10} \text{ s}^{-1}$ . We emphasize that these data represent preliminary measurements, and serve primarily to illustrate a new technique for measuring  $\gamma_\varphi$  in the ballistic regime. In order to obtain better quantitative results, one should measure fluctuation spectra over a broader range of average conductances. Nonetheless, the value for  $\gamma_\varphi$  that we do find provides a reliable order-of-magnitude estimate in a regime where no theoretical predictions are available, and already allows some useful quantitative conclusions to be drawn. For instance, a validity condition for Fermi-liquid theory requires that the effective temperature  $T^*$  of ballistic quasiparticles in the dot must exceed the phase-breaking rate, i.e.,  $kT^* > \hbar\gamma_\varphi$ .<sup>15</sup> From this condition we may infer a rough lower bound for the effective temperature of ballistic electrons in the dot, which is  $T^* \gtrsim 100 \text{ mK}$ . This suggests that the electrons in the dot were not in equilibrium with the measured refrigerator temperature, which was 16 mK.

As the temperature is raised, the phase-breaking rate will increase. From Eq. (1) and the relation  $\alpha \propto \sqrt{\gamma_{\text{eff}}}$  discussed above, one therefore expects at higher temperatures to find larger values of  $\alpha$  (i.e., steeper slopes) in the magnetic frequency spectra of fluctuations. This effect is demonstrated in Fig. 3 for stadium 2 at 20 and 600 mK.

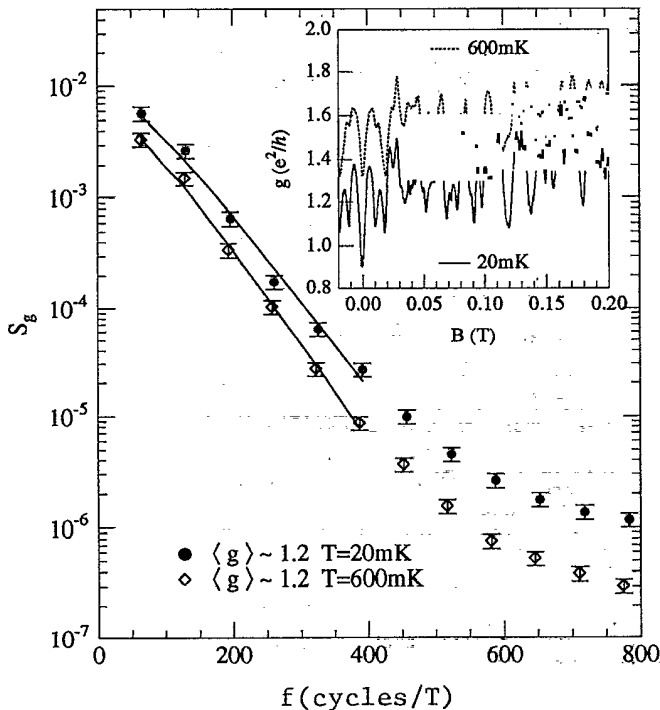


FIG. 3. Averaged magnetoconductance fluctuation power spectra for stadium 2 at mixing-chamber temperatures of 20 mK (filled circles) and 600 mK (open diamonds), measured during the same cooldown without changing gate voltages. Spectra computed as described in the Fig. 2 caption. Solid curves are two-parameter fits to Eq. (3) over the range  $f \sim 64$ –400 cycles/T, giving  $\alpha(20 \text{ mK}) = 0.81$  and  $\alpha(600 \text{ mK}) = 0.89$ . The inset shows part of the fluctuation data used to compute spectra, with 600 mK (dashed line) offset upwards for clarity.

The observed change in the spectral slope in Fig. 3 is rather small:  $\alpha(20 \text{ mK}) = 0.81$  versus  $\alpha(600 \text{ mK}) = 0.89$ , corresponding to an increased rate of phase breaking of roughly 20% at the higher temperature.

#### IV. PHASE-BREAKING RATE FROM CONDUCTANCE FLUCTUATION AMPLITUDE

We next introduce a second method for finding  $\gamma_\varphi$ , this time using the amplitude of the conductance fluctuations rather than their spectral properties. By pinching off the point-contact leads sufficiently by making the gate voltage more negative, one can operate these devices in the regime of partially transmitting single-channel (spin-degenerate) leads, i.e.,  $T_a = T_a^1$ ,  $T_b = T_b^1$ , and  $0 \leq T_a, T_b \leq 1$ . This regime is outside of the semiclassical picture. In the absence of any phase breaking, single-mode leads would imply isolated resonances; however, given a sufficient phase-breaking rate,  $\gamma_\varphi \gtrsim \Delta/\hbar$  ( $\gtrsim 2 \times 10^{10} \text{ s}^{-1}$  in our devices), one may have overlapping resonances together with single-mode leads. Under these conditions, conductance fluctuations—from the single input channel to the single output channel—are essentially equivalent to Ericson fluctuations observed in nuclear scattering experiments.<sup>6,26</sup> We may exploit this similarity by applying the famous Hauser-Feshbach formula<sup>27</sup> from nuclear physics, slightly modified for our present needs.

Consider the fluctuating part  $S_{ab}^{\text{fl}}$  of the scattering matrix element connecting leads  $a$  and  $b$ , defined such that  $\delta g = 2|S_{ab}^{\text{fl}}|^2$  and  $\langle S_{ab}^{\text{fl}} \rangle = 0$ . Heuristically,  $|S_{ab}^{\text{fl}}|^2$  can be thought of as the joint probability for an electron to pass

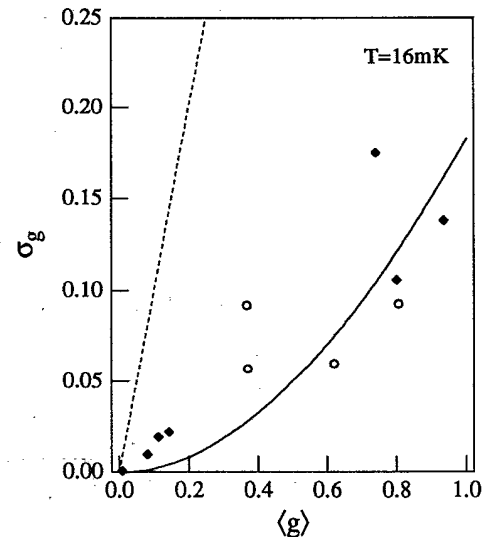


FIG. 4. Fluctuation amplitude  $\sigma_g = \langle \delta g^2 \rangle^{1/2}$  vs average conductance  $\langle g \rangle$ , both in dimensionless units, for stadium 1 and stadium 2 (both filled diamonds) as well as similar circular dots (Ref. 4) (open circles), computed from conductance fluctuation data over a magnetic-field range  $|B| < 0.15 \text{ T}$ . The solid curve is a one-parameter fit of Eq. (5) to all data, giving  $T_\varphi \sim 9$ . The dashed line is speckle behavior  $\sigma_g \sim \langle g \rangle$ , predicted for diffusive systems with single-mode leads in the absence of phase breaking (Ref. 28).

into the dot via lead  $a$ , with probability  $T_a$ , and then pass out of the dot via lead  $b$  without undergoing phase breaking. This will occur with probability  $T_b/(T_a + T_b + T_\varphi)$  assuming the escape time is sufficiently long to allow adequate mixing of electron trajectories.<sup>16</sup> Fluctuations in  $|S_{ab}^n|^2$  are then taken to be exponentially distributed with a standard deviation  $\sigma_S$  equal to the product of these probabilities,<sup>6</sup> giving an amplitude of conductance fluctuations  $\sigma_g \equiv \langle \delta g^2 \rangle^{1/2} = 2T_a T_b / (T_a + T_b + T_\varphi)$ . If we further assume equivalent leads,  $T_a = T_b \equiv T$ , this reduces to the simple form

$$\sigma_g = \frac{\langle g \rangle^2}{\langle g \rangle + T_\varphi/2}. \quad (5)$$

We can use Eq. (5) to find the phase-breaking rate given the fluctuation amplitude  $\sigma_g$  and average conductance  $\langle g \rangle$  through the dot. Figure 4 shows  $\sigma_g$  versus  $\langle g \rangle$  at 16 mK for the two stadium structures, along with data for two similarly fabricated circular structures with the same areas (i.e., same  $\Delta$ , approximately) as the two stadia.<sup>4</sup> The fit to Eq. (5) (solid curve) gives  $T_\varphi \sim 9$ , and hence  $\gamma_\varphi \sim 3 \times 10^{10} \text{ s}^{-1}$ , however, the present data are not sufficiently detailed to argue strongly for the form predicted in Eq. (5). Note that this value is larger by a factor of 2–3 than the value found by the power-spectrum method described above. This difference may be attributable to multimode conduction through the leads even for total lead conductances  $T_a$  and  $T_b$  less than unity. In the multimode case, Eq. (5) overestimates  $\sigma_g$  since fluctuations in the various channels will not add in phase (and

not necessarily in quadrature, either; correlations between fluctuating channels may act to strongly suppress  $\sigma_g$ , as occurs for universal conductance fluctuations in diffusive systems). The value inferred for  $T_\varphi$  from Eq. (5) will, therefore, also be an overestimate due to the assumption of single-mode conduction in each lead.

The two methods introduced for measuring the phase-breaking rates—the semiclassical power-spectrum approach leading to Eqs. (3) and (4) and the Hauser-Feshbach approach leading to Eq. (5)—both lead to experimentally testable predictions. For instance, the quadratic dependence of the fluctuation amplitude at small conductance [from Eq. (5)] ought to be observable and differs from the “speckle”-type behavior  $\sigma_g \sim \langle g \rangle$  predicted for a single-mode two-probe measurement in a diffusive system in the absence of phase breaking (dashed line in Fig. 3).<sup>28</sup> These predictions, as well as detailed measurements of the temperature dependence of ballistic conductance fluctuations, will be investigated in future work.

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